SMGroup

Title: Ternary Digital System: Concepts and Applications

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CHAPTER 2

Ternary Arithmetic Operations

INTRODUCTION

We all are familiar with the arithmetic operations like addition, subtraction, multiplication and division of decimal number. Similar operations can be performed on ternary numbers. The rules for ternary addition, subtraction, multiplication and division are different but these are closely related to binary arithmetic operations.

TERNARY ARITHMETIC

Ternary Addition

Rules for ternary addition are given in table 2.1.

Table 2.1: Ternary addition rules.

Augend	Addend	Sum	Carry	Result
0	0	0	0	00
0	1	1	0	01
0	2	2	0	02
1	1	2	0	02
1	2	0	1	10
2	2	1	1	11

Explanation: Let augend be equal to 0, addend be equal to 0 it gives Sum = 0 + 0 = 0 and no carry generated so carry = 0 as shown in table 2.1. If when augend be equal to 1, addend be equal to 2 we get 1 + 2 = 3.i.e. representation for 3 in ternary is 01 that implies sum = 0 and carry = 1. Similarly when augend be equal to 2, addend be equal to 2 we get 2 + 2 = 4. i.e. representation for 4 in ternary is 11 that implies sum = 1 and carry = 1.

Consider an example

Let Augend A = 64 and Addend B = 48

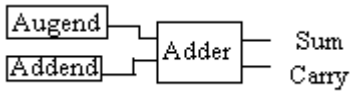
So A + B = 112

Ternary representation for 64 and 48 are 02101 and 01210 respectively.

According to rules given in table 2.1.we can perform addition as:

$$\begin{array}{l} A = 64 = \quad 02101 \\ +B = 48 = \quad 01210 \\ \hline \text{Sum} = 112 = 11011 \end{array}$$

General block diagram for ternary adder is shown below and detail design and logic gate level implementation of circuit for half and full is described in chapter 5.



Block diagram for ternary Adder.

Ternary Subtraction

Rules for ternary subtraction are given in table 2.2.

Table 2.2: Ternary subtraction rules

Minuend	Subtrahend	Difference	Borrow	Result
0	0	0	0	00
0	1	2	1	
0	2	1	1	
1	1	0	0	
1	2	2	2	
2	2	0	0	

Explanation: Let Minuend be equal to 0, Subtrahend be equal to 0 it gives Difference = 0 - 0 = 0 and no Borrow generated so Borrow = 0 as shown in table 2.2. If when Minuend be equal to 1, Subtrahend be equal to 2 we get 1 - 2 = -1.i.e. representation for 3 in ternary is 01 that implies Difference = 0 and borrow = 1. Similarly when Minuend be equal to 2, Subtrahend be equal to 2 we get 2 - 2 = 0. i.e. representation for 0 in ternary is 00 that implies Difference = 0 and borrow = 0.

Consider an example

Let Minuend A = $(64)_{10}$ and Subtrahend B = $(48)_{10}$

So A - B = $(16)_{10}$

Ternary representation for 64 and 48 are $(02101)_3$ and $(01210)_3$ respectively.

According to rules given in table 2.2. we can perform subtraction as:

Let A = 02101 = 64 and B = 01210 = 48, - 48 can be represented as 21012, it is 1's compliment of 48. by adding 1to 1's compliment of 48 we get 3's compliment of 48 i.e.21012 + 1= 21020

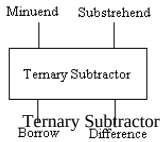
(Details about 1's and 3's compliment is explained in chapter1.)

So A-B = 02101

+ 21020

100121

Discarding left most digit, the answer is $(00121)_3$ which is equivalent to $(16)_{10}$. Block for ternary subtractor is shown in figure 2.2 and details are explained in chapter 5.



Ternary Multiplication

Table 2.3: Rules for ternary multiplications.

Multiplicand	Multiplier	Product	Carry	Result
0	0	0	0	00
0	1	0	0	00
0	2	0	0	00
1	1	1	0	01
1	2	2	0	02
2	2	1	1	11

Explanation: Let Multiplicand be equal to 0, Multiplier be equal to 0 it gives product = 0 and carry = 0 as shown in table 2.3. If when Multiplicand be equal to 1, Multiplier be equal to 2 we get 1 x 2 = 2.i.e. representation for 2 in ternary is 02 that implies product = 2 and carry = 0. Similarly when Multiplicand be equal to 2, Multiplier be equal to 2 we get 2 x 2 = 4. i.e. representation for 4 in ternary is 11 that implies sum = 1 and carry = 1.

Example: Let Multiplicand A = $(15)_{10}$ and B = $(11)_{10}$

So, 15

X 11

$(165)_{10}$

Now $(120)_3$ is ternary representation for $(15)_{10}$ and $(102)_3$ is representation for $(11)_{10}$

i.e. 120 $(15)_{10}$

X 102 $(11)_{10}$

1010

\+ 000x

\+ 120xx

20010 $(165)_{10}$

By applying the rules in table 2.3.3 the result is 20010 which is ternary representation for 165.

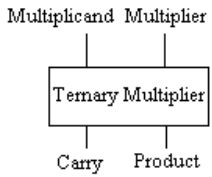


Figure above is for ternary multiplier and detail about design, gate level implementation and circuit is explained in chapter 5.

Ternary Division

It is performed in the same way as binary division. Rules are as below.

1) If Dividend is greater than or equal to divisor, put value to quotient and subtract divisor from dividend.

2) Shift the divisor one place right and perform step 1 up to dividend is smaller than the divisor.

3) If reminder is non zero, repeat step 1 by putting corresponding values to reminder.

4) Repeat until dividend is less than the divisor and quotient is correct. The dividend is the remainder.

Example 1: Let A = $(30)_{10}$ = 1010 is ternary representation and B = $(10)_{10}$ =101is representation for $(10)_{10}$ Therefore A÷B = $(30)_{10}$ ÷$(10)_{10}$ = 1010 ÷ 101= 10 which is ternary representation for $(3)_{10}$

$$
\begin{array}{r}
10 \\
\hline
10\ 1\overline{)1010} \\
-101 \\
\hline
000 \\
-000 \\
\hline
000
\end{array}
$$

Example 2: Let A = $(15)_{10}$ and B = $(3)_{10}$ So A÷B -= $(5)_{10}$
i.e. $(120)_3 \div (10)_3 = (12)_3$

$$
\begin{array}{r}
12 \\
\hline
11\ \overline{)120} \\
-10 \\
\hline
020 \\
-20 \\
\hline
00
\end{array}
$$

SUMMARY

Various number systems that are used in digital circuits, microprocessor, computers etc have been presented. The rules of ternary arithmetic operations like addition, subtraction, multiplication, division are given.

The knowledge of this number system is essential for effective understanding of various ternary circuits.

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