

Geogebra: A vital Bridge Linking Mathematics with Biology and Other Sciences

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Abstract

Our work is part of the world-wide and ongoing efforts of GeoGebra community and other technology communities to increase the rate of change in development of new tools to facilitate and further foster the teaching and learning of mathematics, and its applications. An important part of our research is the use of GeoGebra to demonstrate the Pulfrich stereo-phenomenon.

Our paper is result of the research done with regard to the applications of mathematics in special directions such as: solving problems of physics, biology and astronomy by using technology as a bridge. GeoGebra can be used to perform lots of simulations of physical phenomena from Mechanics discipline to solved computational problems. Our work is part of the passionate work of many GeoGebra users which will result with a very rich fund of GeoGebra virtual tools, of examples and experiences that will be world-widely available for many teachers and GeoGebra software practitioners.

Introduction

Developments in modern mathematics have been driven by a number of motivations that can be categorised into the solution of a difficult problem and the creation of new theory enlarging the fields of applications of mathematics. The solution of a concrete difficult problem is based on the creation of a new mathematical theory. While on the other hand creation of a new mathematical theory may lead to the solution of an old classical problem [1].

Mathematics is one of the key subjects that develops learners to be problem solvers. At present time, the incorporation of Information and Communication Technology (ICT) tools into education and their usage in the teaching and learning mathematics is associated with changes in the global economy.

In the 18th and 19th century mathematical language did not allow much interaction among mathematicians of different fields. In the period 1950's to 1970's mathematicians were concentrated around problems of algebraic topology, algebraic geometry and complex analysis. New concepts, new methods and new powerful mathematical tools were introduced and developed. A great impact was on number and set theory, geometry, topology and partial differential equations which resulted in greater abstraction. During the 20th century were harmonized and unified diverse fields by providing a common language for unification of mathematics which considerably simplified the interaction between mathematicians. The common language became the basis for development of new technical tools for the solution of old problems and the formulation of research programmes. In the period 1930 -1970 there was a divergence within mathematics itself and between mathematics and other applied sciences, and the rapid growth of the scientific community and of the discoveries in physics made much more evident the distinction between pure and applied mathematics. Consequently, the connection of mathematics and other sciences, especially physics was broken. Physicists could not imagine the application of new abstract mathematical concepts (they got interested in solving more concrete problems that could be solved without the application of sophisticated and abstract modern mathematics). On the other side, mathematicians' could not view how physics could assist modern mathematics [1].

The new developments of 1970s, the time when mathematicians returned to more classical topics but on a new level, resulted with a new convergence between mathematics and physics, with deeper connections between modern mathematicians and physicists. The application of modern abstract mathematics in physics led to: new and great discoveries of the 20th Century in the physical sciences, the life sciences and technology, a more developed mathematical language and new powerful mathematical tools, new applications in other applied sciences including computer science and computer technology. The new developments in computer technology, in the discipline of algorithms, mathematical modelling and scientific computing, now-days, are associated with new

discoveries in physics, technology, economics and other sciences. “As science, engineering, government and business rely increasingly on computational simulations, it is inevitable that connections between those sector and the mathematical sciences are strengthened. That is because computational modelling is inherently mathematical” [2].

Ongoing task of mathematicians is the use of modern mathematical tools to solve deep classical problems left by the previous generation of mathematicians.

Traditionally, the applications of mathematics were seen in physics and the tools used were analysis and differential geometry. In the last part of the 20th century researchers in many other sciences understood that the tools of mathematical analysis and differential geometry were no longer adequate. They needed serious and specialized mathematical tools; for instance, a biologist needed the tools of graph theory to understand the genetic code. Because of widespread of the tools of computer technology, the tools of the computer science are more useful than the tools of classical mathematics with regard to the issues of information content. Physics discrete systems are in need of combinatorial tools and statistical mechanics needs the tools of graph and probability theories. Traditionally, the science of economics used applied mathematics toolbox but now, economics utilises sophisticated mathematics in operations research such as linear programming, integer programming and other combinatorial optimisation models [3].

Developments in computer technology are associated with vast and rapid developments in the areas of algorithms, databases, formal languages, as well as cryptography and computer security. The main topics in number theory seemed to be part of the purest mathematics, but now belong to the core of mathematical cryptology and computer security. By the end of the 20th Century, mathematicians understood the need to fill the gaps within mathematics and they challenged to open up more for other disciplines and to foster the beliefs and culture of inter-discipline research. This interaction will be further strengthened in the 21st Century.

“The challenges of theoretical computer science itself are in fact quite mathematical, and the fields of scientific computing and machine learning sit squarely at the interface of the mathematical sciences and computer science (with insight from the domain of application, in many cases)” [2]. An imperative task for mathematics is to enrich other scientific disciplines, and the efforts are not lacking. The mathematicians need to interact with the world around them. Similar efforts are undertaken in other scientific communities. Mathematicians and theoretical physicists are working to bridge the gap between quantum theory and Einstein’s general theory, which are mutually incompatible. The collaborative research is supported by various efforts undertaken to train a new generation of interdisciplinary mathematicians and scientists. There are inter-disciplinary programmes in some universities of developed countries where mathematics and other science students work in joint projects to prepare graduates for the new approaches and practices in their career. Today, almost every science needs people who are mathematically equipped. The science of engineering and medicine require people that know mathematics and statistics, also, other fields that look far of mathematics need such people. All the fields of research need specialized in mathematics people to be active part of the interdisciplinary teams. Without cooperation of people

specialized in different fields a certain research will take a very long time or the results achieved will not thoroughly be true or reliable. Another reason is that the mathematical science researchers not only create the tools that are translated into applications elsewhere but they are also the creative partners who can adapt mathematical sciences results appropriately for different problems. The researchers use the great advances in computing and data collection to investigate more complex phenomena and do more precise analyses. “Computational simulation now guides researchers in deciding which experiments to perform, how to interpret experimental results, which prototype to build, which medical treatments might work, and so on” [2].

ICT is making the greatest impact on education throughout the world. It provides a great opportunity to bridge the knowledge gap between different fields of scientific activity, starting with scientific literacy, training of teachers, involvement in joint projects and so on. It offers opportunities for self-learning, distance and modular learning as well as a powerful tool for self-motivation. It is a powerful teaching and learning resource. ICT has the potential to challenge the fundamental concept of traditional classroom learning. It provides different opportunities for group learning and individual learning. A great success has been in bringing the environment closer to the learner.

Geogebra–A Very Useful, Attractive and Developing Technology

GeoGebra is dynamic mathematics open source software for learning and teaching mathematics in schools. It was developed by Markus Hohenwarter [4] and an international team of programmers. They, followed by many other GeoGebra specialists, have done a brilliant work. Since the time that GeoGebra was introduced are done a great number of progressive steps by many mathematics teachers and lecturers throughout the world. GeoGebra combines geometry, algebra, statistics and calculus. GeoGebra provides the basic features of Computer Algebra System to bridge gaps between Geometry, Algebra and Calculus. The software links the geometric constructions shown in Geometry window to the analytic equations and coordinates representations and graphs shown in Algebra window.

The default GeoGebra view is consisted of the Algebra View, the Graphics View, the Input Bar, and the Tool Bar. There are two ways to construct an object in GeoGebra: use of the Tools in the Toolbar, or use of the corresponding Command entered in the Input Bar. In every case an object is constructed, its algebraic representation appears in Algebra View, whereas its geometrical representation appears in Graphic View. It is this dual representation of objects – visually in the Graphics View and Algebraically in the Algebra View –which makes GeoGebra so powerful.

We can do geometric constructions on the drawing pad of the graphics window and, on the other hand, we can directly enter algebraic input, commands, and functions into the Input field by using the keyboard. To do geometric constructions are used the main virtual tools, which are found in the set of the toolboxes that require from the user to open them, select, activate and use during the construction process. In the toolboxes are found the virtual tools with their names linked with their functions like: New point, Move, Line through two points, Segment between two points etc., alongside is their picture as well. There are also buttons like: Delete object, Move, drawing pad, Zoom in / Zoom out, Undo / Redo buttons etc...

The number of commands that GeoGebra offers is greater than the number of geometry tools.

GeoGebra environment is very pleasant and attractive because it has game features. The virtual tools of GeoGebra can be easily used and played with by anyone of the whole school system. According to Hohenwarter and Preiner [5], GeoGebra appears to be friendly software that can be operated intuitively and does not require advanced skills to get started.

Based on the reports and lessons given in different GeoGebra conferences and other GeoGebra resources is found that the use of GeoGebra fosters students' perseverance, curiosity, inductive attitudes, and inclination to seek accuracy and rigor in geometric learning tasks. Also, are found complex and effective interactions between computer technology, mathematics, and the classroom environment. GeoGebra provides a very comfortable and fruitful environment for research work. This environment provides a higher quality education for students and the capable teachers who implement creative learning environments with technology for the purpose of maximizing the students' learning success that are desperately needed for today and the future [6]. An international community of GeoGebra users is daily growing by the mean of the online GeoGebra Wiki and global and local professional conferences. They are actively addressing traditional problems in mathematics education and developing new pedagogical and theoretical issues on mathematics teaching and learning process, while taking advantage of both technological and theoretical inventions. The use of the computational tool of GeoGebra for teaching functions and their applications in the construction of mathematical models that explain certain physical phenomena is a learning experience with teachers and students where the application of GeoGebra is made to model the real problem in a mathematical problem. GeoGebra is a GNUed software package for mathematics visualization. GNU, which stands for Gnu's Not Unix, is the name for the complete Unix-compatible software system which is given away free to everyone who can use. The home for Geogebra applications is <http://www.geogebra.org>.

Desmos online graphing calculator is another technology. It is a free, browser-based tool that allows users to create, share, and explore math. The online calculator can be easily used by the students to input expressions that plot or graph instantaneously, save and share their work with other social and scientific networks. It is a very beneficial tool for class demonstrations by creating "sliders" to show how changing an equation's parameter affects a graph's shape. The Desmos online graphing calculator provides the best tools for learning and the right environment where everyone can learn and enjoy math and create useful products. It can be found at the address: <https://www.desmos.com/calculator>.

Visualization of Pulfrich Phenomenon with GeoGebra

The Pulfrich phenomenon or effect is a psychophysical perception related to the apparent spatial disparity between the two eyes when the sight of one of the eyes is purposely passed through a filter. The effect was observed in an experiment with a swinging pendulum that was first performed in 1922 by a German physicist called Carl Pulfrich. The classical demonstration of the Pulfrich Effect is as following: The pendulum is swung back and forth and its movement is observed with both eyes. Care must be shown that the movements of the pendulum stay in the same plane parallel to the viewer's forehead, which is

making the observation. The meaning of this is that the swinging be in the plane which is vertical to the observer's sight rays (the trace of the pendulum is a circular part lied in this plane), in other words: not in a way that the pendulum shifts near and far of this plane (in this case its trace is a circle that cuts the plane). When viewed normally with both eyes, the pendulum just seems to swing back and forth. When a filter is placed in front of one eye, the pendulum suddenly seems to be swinging along an ellipse, parallel to the floor. In other words, the pendulum should appear to be tracing an oval path. This is an optical illusion. Weakening the light equally to both eyes does not cause the illusion [7] (Figure 1).

The Pulfrich Effect, which happens when a filter is placed in front of one eye, is an optical illusion in which an object which is moving in a plane parallel to the viewer's forehead seems to move out of the plane mentioned above. The trace of the pendulum looks as if approaching (or receding from) the viewer. To get this illusion, one has to weaken the light reaching one eye, using a sunglass lens or a smoked glass. The illusion occurs only when the moving object is being viewed with both eyes and a filter is placed in front of one eye. Anyone can try out this effect with a pendulum, although Pulfrich was never able to observe the phenomenon because he was blind in one eye. He explained the phenomenon without observing it. This is the reason why this phenomenon bears his name.

Explanation of Pulfrich phenomenon

The placement of the filter in front of one eye, usually this is a neutral density filter [8] is "causing a time delay for the neural signal in the eye." Therefore, the perceived position of the swinging pendulum in the filtered eye lags" slightly behind that in the other eye for a given moment, creating this way a spatial offset between what is perceived in the two eyes" [8]. In the case, when the right eye is covered by the filter: when the pendulum swings from left to right, the "spatial offset corresponds to a binocular disparity that shifts the perceived depth of the pendulum towards the observer" and, when the pendulum swings from right to left, the "spatial offset corresponds to a binocular disparity that shifts the perceived depth of the pendulum away from the observer" [8]. As said earlier, the target which is oscillating at eye level in a front parallel plane appears to rotate out of its oscillating plane when a coloured filter is placed in front of one of the observer's eyes. The distance (the depth of displacement) of the apparent target from the parallel plane increases as the difference of binocular retinal illumination is increased. Pulfrich (following a suggestion by Fertsch)

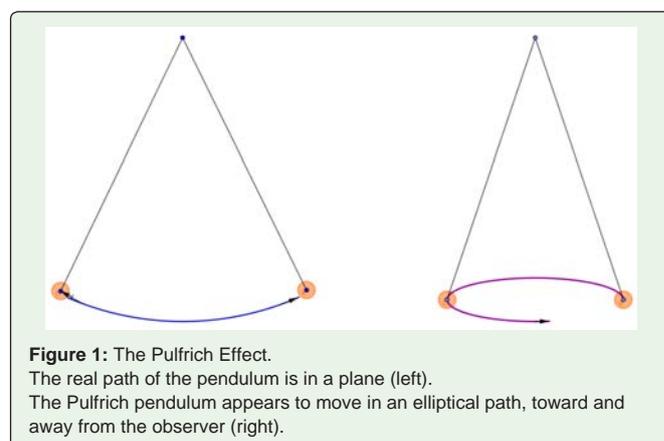


Figure 1: The Pulfrich Effect.

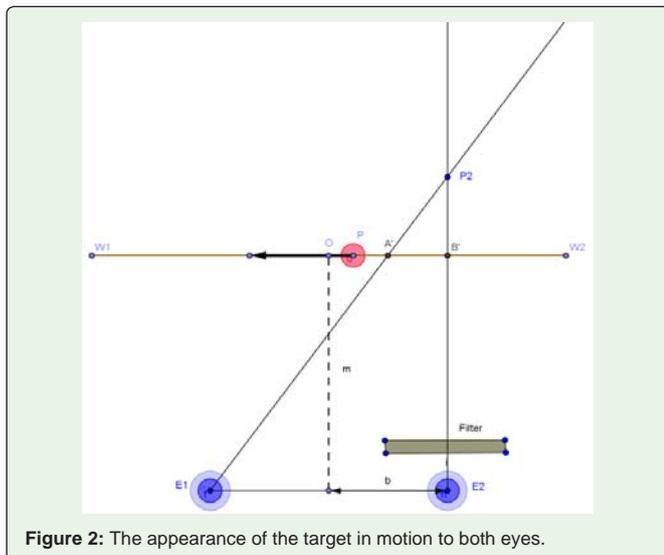
The real path of the pendulum is in a plane (left).

The Pulfrich pendulum appears to move in an elliptical path, toward and away from the observer (right).

accounted for these depth displacements in terms of a difference in the hypothesized visual latent periods of the two eyes. The visual latent period of each eye was assumed to be a reciprocal function of the level of retinal illumination. Thus, the eye covered with the filter presumably “signals” a position of the oscillating target that lags behind the position that was signalled by the uncovered eye. Hence, at any given moment, synchronous binocular signals are provided by pairs of non-corresponding retinal points in the two eyes, and the magnitude of the stereo-effect theoretically depends on the amount of the retinal disparity produced as a consequence of the difference in the visual latent periods of the two eyes.

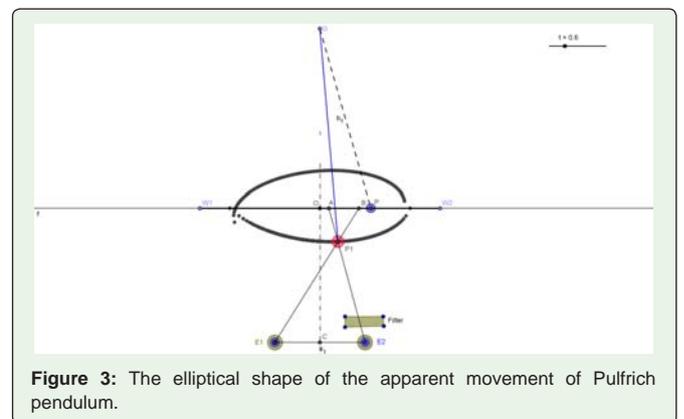
Joel D. Brauner and Alfred Lit [9], in a long-range research program, have studied the effects of conditions of illumination of visual latency, measured by several different monocular and binocular experimental procedures, in which the magnitudes of the far and near displacements of the Pulfrich stereo-phenomenon were measured at various levels of illumination (1975) [10]. A simple model of the appearance of the target in motion to both eyes of an observer would be as in figure 2. The object is swinging from right to left. The distance by which the covered eye’s view lags the true position of pendulum is geometrically represented by the length of segment PB' , whereas the distance by which the bright eye’s view lags the true position of pendulum is geometrically represented by the length of segment PA' . It is clear that $PB' > PA'$. As explained, the covered eye (E_2) produces a slightly delayed signal with respect to the bright eye (E_1). How the human brain interprets the pair of images out of plane depends upon which way the object is moving. The movement from right to left (Figure 2) results in the object appearing further (position P_2) than it actually is (position P), while the opposite movement makes it seem nearer.

The size of these shifts depends upon how fast the object is moving. A swinging pendulum moves at its fastest in the middle of the swing (the minimum point of the circular trace), and comes to a momentary stop at the ends, where it changes direction. Consequently, when viewing a pendulum with one eye, covered with a filter, the apparent distance shift will be greatest in mid-swing and zero at the ends of the swing. The result is that the pendulum seems to follow an elliptical path.



It is known that the kinetic energy and velocity of the pendulum are maximized at the equilibrium point and minimized when it changes the direction which is the maximum angular shift of the pendulum from the equilibrium point. Using the equations,

$E = 1/2 mv^2 = mgh$ and $h = L - L \cos(\theta_{max})$, at the equilibrium point, is found that $v = \sqrt{2gL(1 - \cos\theta_{max})}$. As can be seen, the shape of the ellipse depends on the speed of the object which is a function of the string’s length and of the maximum angular shift. Much more the shape of the ellipse depends on the individual physical and optical features of the observer and the kind of lens/filter that is put in front of one eye. So, the shape of the ellipse is an issue of broader scope and can be confirmed only by experimental results. It is not our aim to talk more about this. In Figure 3, the apparent path is almost coincident with the ellipse of equation $x^2/36 + y^2/7 = 1$. Our purpose is not to calculate the time lag or the distances of the stimulation points of the two eyes (designated by A and B) from the centre O (as shown in Figure 3) but to demonstrate the apparent movement of the pendulum using GeoGebra software. We have shown a short path (considering it linear) of the target (pendulum) in its plane of oscillation which is denoted by W_1W_2 . The distance of the plane of oscillation is measured from the midpoint of the line E_1E_2 where, E_1 represents the left eye and E_2 represents the right eye. The filter is placed in front of the right eye. Point P represents the swinging object and GP (dotted segment) represents its string. The point P_1 (red colour object) represents a near position in which the oscillating target, at point P, is localized by the observer as the target moves from left to right. The point P_2 represents a far position in which the oscillating target, denoted again by P, is localized by the observer as the target moves from right to left. The point P_1 is appeared when the target moves from left to right, whereas the point P_2 is appeared when the target moves from right to left. The intersection points of the lines of sight with the line W_1W_2 are designated by A and B, when the target moves from left to right, and these points theoretically mark the respective positions in the path of the oscillating target at which the start of stimulation occurred in each of the two eyes. When the target moves from right to left these points are designated by A' and B' , respectively. Thus, when the target is moving from left to right and appears to be located at the near position P_1 towards which the eyes are converged, the start of stimulation for the left eye occurs when the target (at P in the Figure) is located at point B, and the start of stimulation for the right eye, covered by the filter, occurs when the target is located at point A, a bit farther behind in its path. The time taken for the target to move from A to P represents the magnitude



of the visual latent period of the right eye, and the time taken for the target to move from B to P represents the slightly shorter visual latent period of the left eye.

Application of GeoGebra in Biomechanics

The applet of demonstration of the Pulfrich effect is created using GeoGebra software. We are showing here the details of how the applet is created. The Applets pages are created with GeoGebra software and they can be downloaded from the wiki.

Procedure

Pendulum swinging from left to right (the image of the pendulum appears nearer to the observer)

1. Open a new GeoGebra window.
2. Construct a slider with parameter t ; Interval: $(-6,18)$, increment 0.1; Animation: speed 1 to get average movement of the pendulum, repeat: increasing.
3. Construct the segment W_1W_2 and the points $E_1 E_2$ representing the eyes.
4. Define the positions of points A and B in the segment W_1W_2 : enter in the Input field Function $[x,-6,6] = h(x)$ corresponding to A, enter in the Input field Function $[1 / 3 \sqrt{36 - x^2} + x, -6, 6] = p(x)$ corresponding to B. The second function is an elliptical one and is purposely chosen in that form in order that when tried the simulation get an oval path. Construct points $A = (h(t),0)$ and $B = (p(t),0)$ and $P = (p(t)+0.8,0)$. Point P represents the true position of the pendulum and must be the guiding point for A and B. The length of segment AB is $AB = p(t) - h(t) = 1 / 3 \sqrt{36 - t^2}$, which takes the maximum value for $t = 0$ corresponding to the position of the pendulum at the equilibrium centre (the highest speed) where the length of segment AB is maximum (the difference between the time signals produced by the object in the two eyes is maximum).
5. Construct the segments E_1B and E_2A and their intersection point P_1 (this is the nearer image).
6. Test the path of the point P_1 by right-clicking on point P_1 and selecting Trace On in the displayed table and, after this, selecting in the slider the option Animation On.

Pendulum swinging from right to left (the image of the pendulum appears far away of the swinging plane with regard to the observer):

1. Define the positions of points A' and B' in the segment W_1W_2 : enter in the Input field Function $[-x+12,6,18] = g(x)$ corresponding to A', enter in the Input field Function $[1 / 3 \sqrt{36 - (x - 12)^2} - x + 11.3, 6.8, 18] = s(x)$ corresponding to B'. The second function is an elliptical one and is purposely chosen in that form in order that when tried the simulation get an oval path. Construct points $A' = (g(t),0)$ and $B' = (s(t),0)$ and $P' = (g(t)-0.8,0)$. Point P' represents the true position of the pendulum and must be the guiding point for A' and B' during the swinging.
2. Construct the segments E_1A' and E_2B' and their intersection point P_2 (this is the further image).
3. Test the path of the point P_2 by right-clicking on point P_2 and selecting Trace on in the displayed table and, after this, selecting in the slider the option Animation on.

Now, you can watch the two phases of the pendulum swinging, even with repetitions. The path is almost elliptical.

For our applet go to the link, <http://www.geogebra.org/student/m30375> and watch the simulation.

Possible Project

We hope that this paper will draw the attention of many oculists and vision therapists (persons who are skilled in vision therapy). It is known that there are people who have visual problems derived from the weakening of the eye muscles. Depending on the age, the eye muscles become weaker as the age grows. There are vision problems that need surgical intervention or treatment, but there are vision problems that need non-surgical treatment. Such are: lazy eye, crossed eyes, double vision, reading and learning disabilities, and convergence insufficiency. The second type of vision problems, we think, can be treated using the simulation of Pulfrich effect. This is a home-based therapy, consisted of computer-based exercises, to vitalize the eye muscles by watching on computer screen the simulation of the Pulfrich effect, which is downloaded via the link above. By watching the movement of pendulum, the eye muscles are put in motion. It is so easy, anyone can do, it is a computer game. Our opinion is that this kind of therapy will recover or strengthen the eye muscles.

""There are many other applications of math in Biology, Physics and Engineering where GeoGebra software is used. Many simulations are linked with mechanics, optics and dynamics etc. There are many calculations that are performed using GeoGebra: calculation of centroid, of pressure and so on. Also, there is teaching about the principles and the laws in micro world and macro world, plus virtual laboratories. There is no space to treat them here in this paper.

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